Solutions - Homework 1

(Due date: Sep. 26th (008), Sep. 27th (010) @ 11:59 pm)

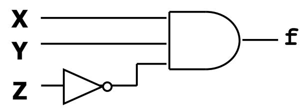
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (28 PTS)

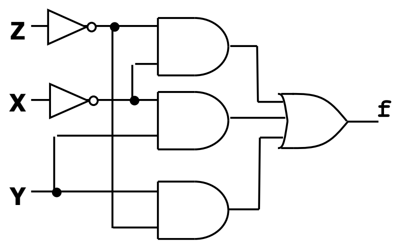
a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (15 pts)

✓ $f = \overline{y(z+\bar{x}) + \overline{yx}}$ ✓ $f = \prod(M_1, M_4, M_5, M_7)$ ✓ $f(A,B,C) = \overline{AB\bar{C} + (\overline{C \oplus A})B}$

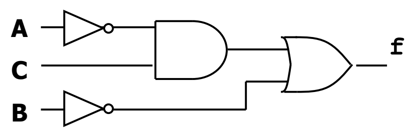
✓ $f(x,y,z) = \overline{y(z+\bar{x}) + \overline{yx}} = \overline{YZ + Y\bar{X} + \bar{X} + \bar{Y}} = \overline{YZ + \bar{X} + \bar{Y}} = \overline{\bar{X} + (\bar{Y}+Y)(\bar{Y}+Z)} = \overline{\bar{X} + \bar{Y} + Z} = XY\bar{Z}$



✓ $f(X,Y,X) = \prod(M_1, M_4, M_5, M_7) = \sum(m_0, m_2, m_3, m_6) = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}YZ + XY\bar{Z} = \bar{X}\bar{Z} + \bar{X}YZ + XY\bar{Z}$
$= \bar{X}(\bar{Z} + YZ) + XY\bar{Z} = \bar{X}(\bar{Z} + Y) + XY\bar{Z} = \bar{X}\bar{Z} + \bar{X}Y + XY\bar{Z} = \bar{X}Y + \bar{Z}(\bar{X} + XY) = \bar{X}Y + \bar{Z}(\bar{X} + Y)$
$= \bar{X}Y + \bar{Z}\bar{X} + \bar{Z}Y$



✓ $f(A,B,C) = \overline{AB\bar{C} + (\overline{A \oplus C})B} = \overline{AB\bar{C} + (AC + \bar{A}\bar{C})B} = \overline{AB(\bar{C} + C) + \bar{A}\bar{C}B} = \overline{AB + \bar{A}\bar{C}B} = \overline{B(A + \bar{A}\bar{C})}$
$= \overline{B(A+\bar{C})} = \bar{B} + \overline{(A+\bar{C})} = \bar{B} + \bar{A}C$



b) Using Boolean Algebra Theorems, prove that: $y(x \oplus z) = (yx) \oplus (yz)$ (5 pts)

$y(x \oplus z) = y(\bar{x}z + x\bar{z}) = y\bar{x}z + yx\bar{z}$
$(yx) \oplus (yz) = \overline{yx}yz + yx\overline{yz} = (\bar{y} + \bar{x})yz + yx(\bar{y} + \bar{z}) = \bar{y}yz + \bar{x}yz + yx\bar{y} + yx\bar{z} = \bar{x}yz + yx\bar{z}$

$\therefore\ y(x \oplus z) = (yx) \oplus (yz)$

c) For the following Truth table with two outputs: (8 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS). (4 pts)
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums. (3 pts)

x	y	z	f_1	f_2
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Sum of Products

$f_1 = \bar{x}\bar{y}\bar{z} + x\bar{y}z + xyz$

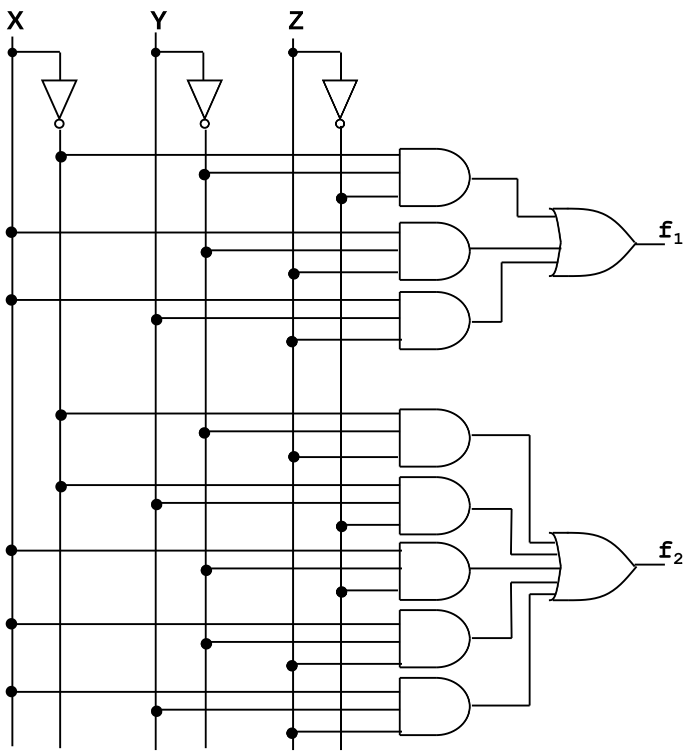
$f_2 = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z + xyz$

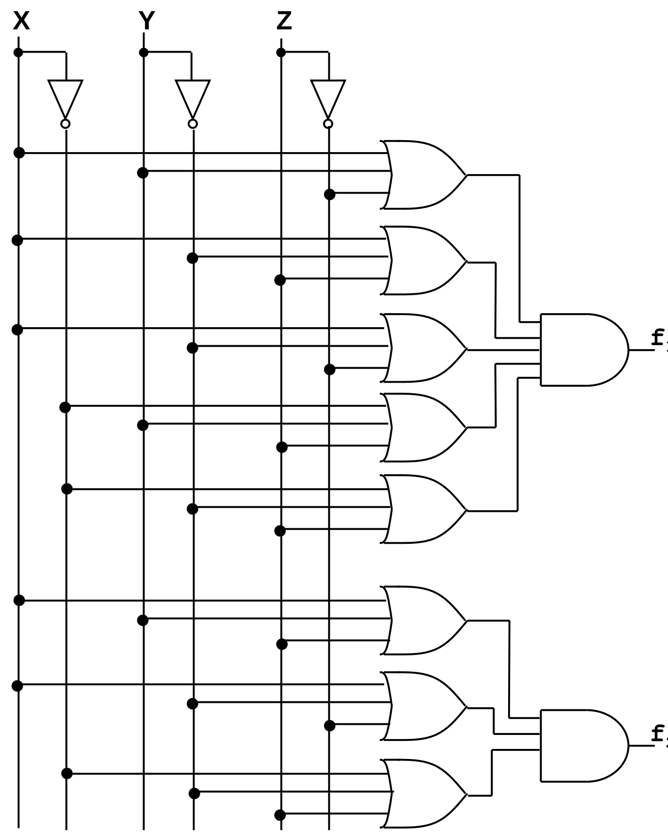
Product of Sums

$f_1 = (x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + y + z)(\bar{x} + \bar{y} + z)$

$f_2 = (x + y + z)(x + \bar{y} + \bar{z})(\bar{x} + \bar{y} + z)$

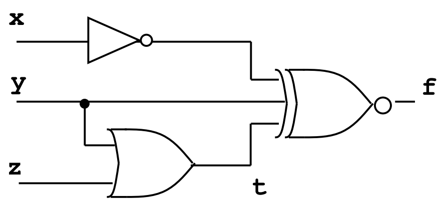
Minterms and maxterms: $f_1 = \sum(m_0, m_5, m_7) = \prod(M_1, M_2, M_3, M_4, M_6)$.
$f_2 = \sum(m_1, m_2, m_4, m_5, m_7) = \prod(M_0, M_3, M_6)$.





PROBLEM 2 (26 PTS)

a) Construct the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).
Note that $a \oplus b \oplus c = (a \oplus b) \oplus c = a \oplus (b \oplus c) = b \oplus (a \oplus c)$

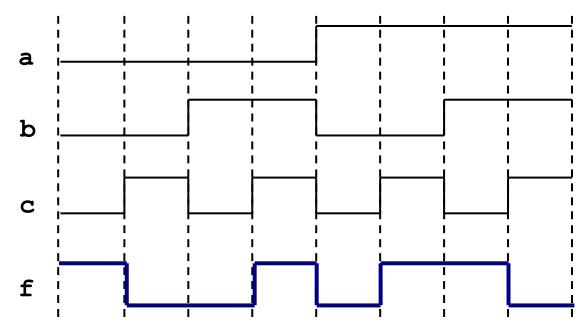


x	y	z	t	f
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

$f = \overline{\bar{x} \oplus y \oplus (y+z)} = \overline{\bar{x} \oplus (\bar{y}z)} = \bar{x}\bar{y}z + x\overline{\bar{y}z} = \bar{x}\bar{y}z + xy + x\bar{z}$

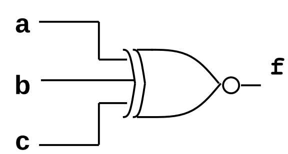
b) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code (using VHDL signals is optional). (8 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
 port ( a, b, c: in std_logic;
        f: out std_logic);
end circ;

architecture st of circ is

begin
   f <= not (a xor b xor c);
end st;
```



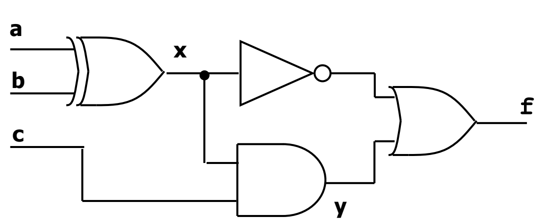
a	b	c	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

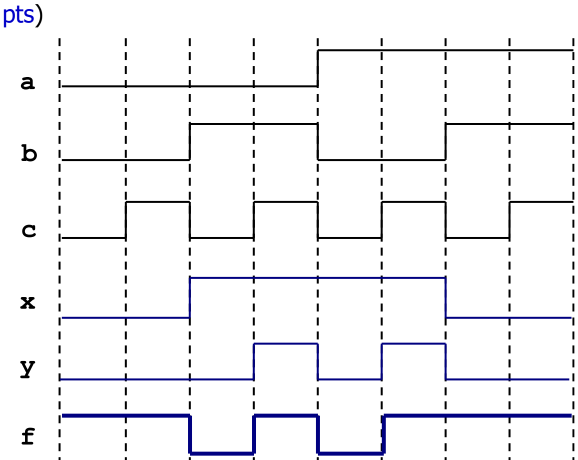
c \ ab	00	01	11	10
0	1	0	1	0
1	0	1	0	1



$f = \bar{a}\bar{b}\bar{c} + \bar{a}bc + ab\bar{c} + a\bar{b}c = \bar{a}(\bar{b}\bar{c} + bc) + a(b\bar{c} + \bar{b}c) = \bar{a}(\overline{b \oplus c}) + a(b \oplus c) = \overline{a \oplus (b \oplus c)} = \overline{a \oplus b \oplus c}$

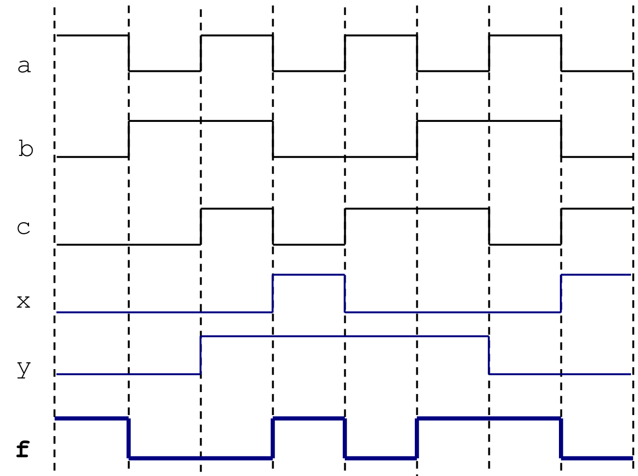
c) Complete the timing diagram of the following circuit: (5 pts)





d) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (7 pts)

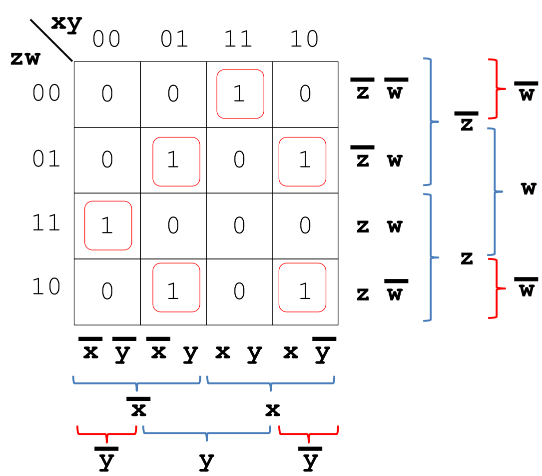
```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;

begin
  f <= (not y) xnor a;
  x <= a nor b;
  y <= x xor c;
end st;
```



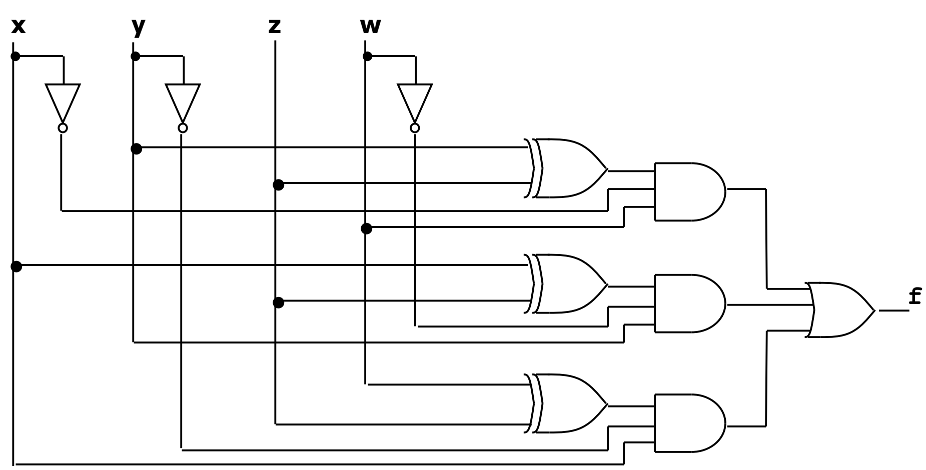
PROBLEM 3 (10 PTS)

- Complete the truth table for a circuit with 4 inputs x, y, z, w that activates an output ($f = 1$) when the number of 1's in the inputs is equal than the number of 0's. For example: If $xyzw = 1001 \rightarrow f = 1$. If $xyzw = 1011 \rightarrow f = 0$.
- Provide the Boolean equation for the output f using the minterms representation.
- Sketch the logic circuit.

x	y	z	w	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



$f = \bar{x}\bar{y}zw + \bar{x}y\bar{z}w + \bar{x}yz\bar{w} + xy\bar{z}\bar{w} + x\bar{y}\bar{z}w + x\bar{y}z\bar{w} = \bar{x}w(y \oplus z) + y\bar{w}(x \oplus z) + x\bar{y}(z \oplus w)$

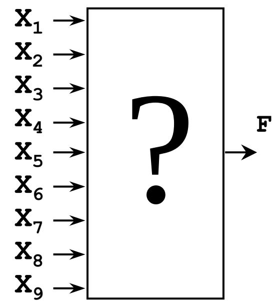


PROBLEM 4 (11 PTS)

- Tic-tac-toe game (3-by-3 grid of squares). The players alternate turns. Each player chooses a square and places a mark in a square (one player uses **X** and the other **O**). The first player with three marks in a row, column, or diagonal wins the game.
- Design a digital circuit for an electronic tic-tac-toe that indicates the presence of a winning pattern for player **X**. The circuit has 9 inputs (X_1 to X_9) and an output **F**.
  - ✓ Inputs X_1 to X_9: A value of '1' indicates that the player marked the corresponding position with an **X**. A value of '0' indicates that the other player marked that position.
  - ✓ **F** ='1' if a winning pattern is present and **F** ='0' otherwise.
  - ✓ Example: if X_1=1, X_2=0, X_3=1, X_4=0, X_5=1, X_6=0, X_7=1, X_8=0, X_9=1, then **F**=1.

- Provide the Boolean expression for **F**. The 9 inputs (X_1 to X_9) are arranged in the following pattern:
  Tip: Note that if there are three 1's in a winning pattern, the value of the remining 6 positions is irrelevant.

X_1	X_2	X_3
X_4	X_5	X_6
X_7	X_8	X_9



$f = 1$ when there are three 1's in a line. Note that there can be more than three 1's before we find three 1's in a line. So, we make $f = 1$ when a winning pattern is present regardless of the values of the remaining variables:

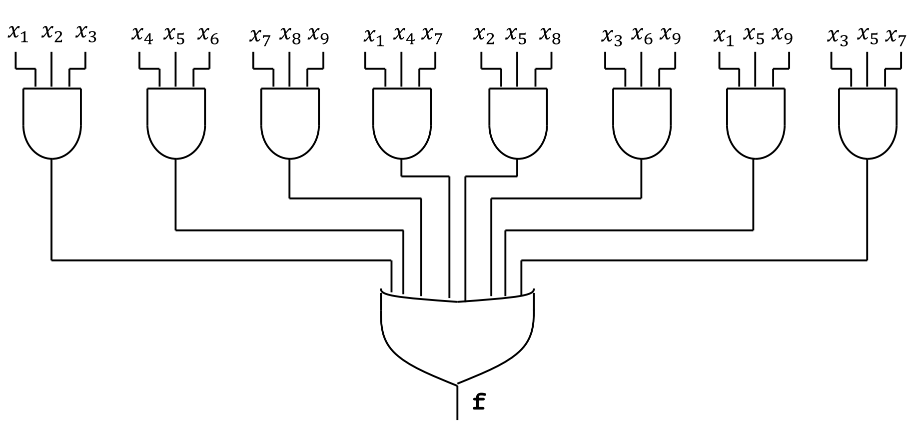
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	f
1	1	1	X	X	X	X	X	X	1
X	X	X	1	1	1	X	X	X	1
X	X	X	X	X	X	1	1	1	1
1	X	X	1	X	X	1	X	X	1
X	1	X	X	1	X	X	1	X	1
X	X	1	X	X	1	X	X	1	1
1	X	X	X	1	X	X	X	1	1
X	X	1	X	1	X	1	X	X	1
All remaining cases									0

First row: $x_1x_2x_3.(all\ permutations\ for\ x_4x_5x_6x_7x_8x_9) = x_1x_2x_3.h(x_4,x_5,x_6,x_7,x_8,x_9)$
Where: $h(x_4,x_5,x_6,x_7,x_8,x_9) = \sum m(0,1,\dots,63) = 1$ (the sum of all the minterms of a 6-variable function is always 1).
Thus, Then first row is just: $x_1x_2x_3$

The same technique is applied for every row.

$\rightarrow f = x_1x_2x_3 + x_4x_5x_6 + x_7x_8x_9 + x_1x_4x_7 + x_2x_5x_8 + x_3x_6x_9 + x_1x_5x_9 + x_3x_5x_7$

- ✓ Sketch the logical circuit resulting from the Boolean equation for **F**.

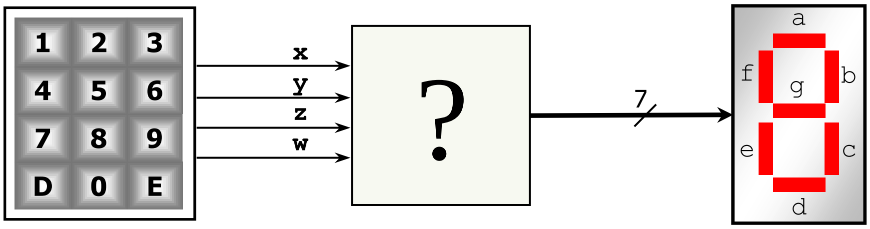


PROBLEM 5 (25 PTS)

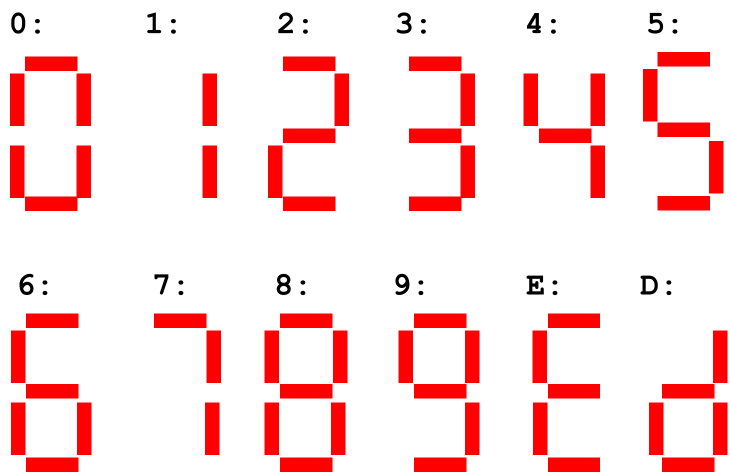
- A numeric keypad produces a 4-bit code as shown below. We want to design a logic circuit that converts each 4-bit code to a 7-segment code, where each segment is an LED. The LEDs are lit with a logical '0' (negative logic). The inputs are active high (or in positive logic).

✓ Complete the truth table for each output (a, b, c, d, e, f, g). (4 pts)

✓ Provide the simplified expression for each output (a, b, c, d, e, f, g). Use Karnaugh maps for a, b, c, d, e and the Quine-McCluskey algorithm for f, g. Note that it is safe to assume that the codes 1100 to 1111 will not be produced by the keypad.



Value	X	Y	Z	W	a	b	c	d	e	f	g
0	0	0	0	0							
1	0	0	0	1							
2	0	0	1	0							
3	0	0	1	1							
4	0	1	0	0							
5	0	1	0	1							
6	0	1	1	0							
7	0	1	1	1							
8	1	0	0	0							
9	1	0	0	1	0	0	0	0	1	0	0
E	1	0	1	0							
D	1	0	1	1							
	1	1	0	0							
	1	1	0	1							
	1	1	1	0							
	1	1	1	1							



Value	X	Y	Z	W	a	b	c	d	e	f	g
0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	1	1	0	0	1	1	1	1
2	0	0	1	0	0	0	1	0	0	1	0
3	0	0	1	1	0	0	0	0	1	1	0
4	0	1	0	0	1	0	0	1	1	0	0
5	0	1	0	1	0	1	0	0	1	0	0
6	0	1	1	0	0	1	0	0	0	0	0
7	0	1	1	1	0	0	0	1	1	1	1
8	1	0	0	0	0	0	0	0	0	0	0
9	1	0	0	1	0	0	0	0	1	0	0
E	1	0	1	0	0	1	1	0	0	0	0
D	1	0	1	1	1	0	0	0	0	1	0
	1	1	0	0	X	X	X	X	X	X	X
	1	1	0	1	X	X	X	X	X	X	X
	1	1	1	0	X	X	X	X	X	X	X
	1	1	1	1	X	X	X	X	X	X	X

a

zw \ xy	00	01	11	10
00	0	1	X	0
01	1	0	X	0
11	0	0	X	1
10	0	0	X	0

b

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	1	X	0
11	0	0	X	0
10	0	1	X	1

c

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	0	X	0
11	0	0	X	0
10	1	0	X	1

d

zw \ xy	00	01	11	10
00	0	1	X	0
01	1	0	X	0
11	0	1	X	0
10	0	0	X	0

e

zw \ xy	00	01	11	10
00	0	1	X	0
01	1	1	X	1
11	1	1	X	0
10	0	0	X	0

$a = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + xzw$
$b = y\bar{z}w + yz\bar{w} + xz\bar{w}$
$c = \bar{y}z\bar{w}$
$d = \bar{x}\bar{y}\bar{z}w + y\bar{z}\bar{w} + yzw$
$e = y\bar{z} + \bar{z}w + \bar{x}w$

- $f = \sum m(1,2,3,7,11) + \sum d(12,13,14,15)$

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
1	m_1 = 0001 ✓ m_2 = 0010 ✓	**$m_{1,3}$ = 00-1** **$m_{2,3}$ = 001-**		We can't combine any further, so we stop here
2	m_3 = 0011 ✓ m_{12}= 1100 ✓	$m_{3,7}$ = 0-11 ✓ $m_{3,11}$ = -011 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{3,7,11,15}$ = --11** ~~$m_{3,11,7,15}$ = --11~~ **$m_{14,15,12,13}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~	
3	m_7 = 0111 ✓ m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{7,15}$ = -111 ✓ $m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$$f = \bar{x}\bar{y}w + \bar{x}\bar{y}z + zw + xy$$

Prime Implicants		Minterms 1	2	3	7	11
$m_{1,3}$	$\bar{x}\bar{y}w$	**X**		X		
$m_{2,3}$	$\bar{x}\bar{y}z$		**X**	X		
$m_{3,7,11,15}$	zw			X	**X**	X
$m_{14,15,12,13}$	xy					

$$\rightarrow f = \bar{x}\bar{y}w + \bar{x}\bar{y}z + zw$$

- $g = \sum m(0,1,7) + \sum d(12,13,14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicants
0	m_0 = 0000 ✓	**$m_{0,1}$ = 000-**		
1	m_1 = 0001 ✓			We can't combine any further, so we stop here
2	m_{12}= 1100 ✓	$m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~	
3	m_7 = 0111 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	**$m_{7,15}$ = -111** $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$$g = \bar{x}\bar{y}\bar{z} + yzw + xy$$

Prime Implicants		Minterms 0	1	7
$m_{0,1}$	$\bar{x}\bar{y}\bar{z}$	**X**	X	
$m_{7,15}$	yzw			**X**
$m_{12,13,14,15}$	xy			

$$\rightarrow g = \bar{x}\bar{y}\bar{z} + yzw$$